1st Class

2023-2024

Discrete Structures

الهياكل المتقطعة

Lecture 1

Logic

1.1 Propositions and Truth Values :

A proposition is declarative statement which is either true or false, but not both. (propositions are sometimes called(**statements**).

Examples: -

1. Triangles have four vertices.

2. 6 + 2 = 4.

3. 5 < 24.

The truth (**T**) or falsity (**F**) of a proposition is called **Truth Value.** Proposition 3 has a truth value of true (T) and propositions 1&2 have truth values of false (F).

* Questions & demands are not propositions, since they can not be declared true or false. Thus the following are not propositions:

4. Keep off the cat.

5. Did you go to party?

6. Don't say that.

Sentences 4 – 6 are not propositions and therefore cannotbe assigned truth values.

* Propositions are denoted using the letters p, q, r , … . Any of these letters may be used to symbolize specific propositions.

Compound proposition:

A compound proposition is statement formed by connecting two or more statement, or by negating a simpleproposition.

1 .2 Logical connectives:

1) Negation: (~)

If p is any proposition, the negation of p denoted by ~p (or p or ¬ p). And it's a proposition which is false when pis true, and true when p is false.

- We can summarize this in a table.

p	~ p
T	F
F	T

2) Conjunction: (And) (Λ)

Let p & q be any two propositions, the compound proposition is called conjunction of p & q. And denoted by (p Λ q) .

The following table gives the truth values of p Λ q :

P	Q	p Λ q
T	T	T
T	F	F
F	T	F
F	F	F

From the table it can be seen that the conjunction p Λ q is true only when p and q are both true. Otherwise the conjunction is false.

3) Disjunction: (or) (V)

Let p & q be any two propositions, compound proposition iscalled disjunction of p &q. And it's denoted by (p V q).

The following table gives the truth value of (p V q) :

P	Q	p V q
T	T	T
T	F	T
F	T	T
F	F	F

From the previous table , one can notice that p V q is true when either or both of it's components are true and it's false otherwise.

4) Conditional Propositions: (→)

The conditional connective (sometimes Called **implication**) is denoted by →. And reed as if p then q, for any two propositions p & q.

The following is the truth table for p → q:

P	q	p → q
T	T	T
T	F	F
F	T	T
F	F	T

Notice that " the proposition " if p then q " is false only when p is true and q is false. i. e , a true statement can not imply a false one .

5) Biconditional Propositions :(↔) (if and only if)

The biconditional connective is denoted by ↔. and expressed by " if and only if …. Then … "The truth table of p ↔ q is,

P	q	p ↔ q
T	T	T
T	F	F
F	T	F
F	F	T

Note that for p ↔ **q** to be true, when p and q must both have the same truth value. i . e, both must be true or both mustbe false.

Examples: -

A B

1. Construct a truth table for (q V p) Λ (~ p V ~ q) .

p	q	~ p	~ q	q V p	(~ p V ~ q)	A Λ B
T	T	F	F	T	F	F
T	F	F	T	T	T	T
F	T	T	F	T	T	T
F	F	T	T	F	T	F

2. Construct a truth table for (~ q Λ p) V (~q V ~ p) Λp.

A B

C

p	q	~ p	~ q	~ q Λ p	(~qV~p)	A VB	C Λ P
T	T	F	F	T	F	F	F
T	F	F	T	T	T	T	T
F	T	T	F	T	T	T	F
F	F	T	T	F	T	F	F

3. Construct a truth table for :

a) ~q →p . b) ~p↔~q . c) p →(qΛr) . d) (~p V q) ↔~r.

a)

p	q	~q	~q→p
T	T	F	T
T	F	T	T
F	T	F	T
F	F	T	F

b)

p	q	~p	~q	~p↔~q
T	T	F	F	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

c)

P	q	r	qΛr	p→(qΛr)
T	T	T	T	T
T	T	F	F	F
T	F	T	F	F
T	F	F	F	F
F	T	T	T	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

d)

p	Q	r	~p	~r	~p Vq	(~p Vq)↔ ~r
T	T	T	F	F	T	F
T	T	F	F	T	T	T
T	F	T	F	F	F	T
T	F	F	F	T	F	F
F	T	T	T	F	T	F
F	T	F	T	T	T	T
F	F	T	T	F	T	F
F	F	F	T	T	T	T

$2^2 = 4 \rightarrow$ (b) , (a)

$2^3 = 4 \rightarrow$ (d) , (c)

Exercises:

1) Draw the truth tables for the proposition :

1. ~p → q. 2. ~q Λ p. 3. (p V q) → (p Λ q) .

4. ~p↔ (p Λ q) .

2) Given the propositions . p , q & r , construct the truth tables for :

1. (p Λ q) → ~r . 2. p Λ (~q V r) . 3. ~ (p V q) ↔ (r V p).